# Exercise 2.4.2

Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails because $f'(x^*) = 0$, use a graphical argument to decide the stability.

$$\dot{x} = x(1 - x)(2 - x)$$

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### Solution

The fixed points occur where $\dot{x} = 0$.

$$x^*(1 - x^*)(2 - x^*) = 0$$

$$x^* = 0 \quad \text{or} \quad 1 - x^* = 0 \quad \text{or} \quad 2 - x^* = 0$$

$$x^* = 0 \quad \text{or} \quad x^* = 1 \quad \text{or} \quad x^* = 2$$

Use linear stability analysis to classify these points.

$$\begin{aligned} f(x) &= x(1 - x)(2 - x) \\ &= 2x - 3x^2 + x^3 \end{aligned}$$

Differentiate $f(x)$.

$$f'(x) = 2 - 6x + 3x^2$$

As a result,

$$\begin{aligned} f'(0) &= 2 > 0 && \Rightarrow && x^* = 0 \text{ is an unstable fixed point.} \\ f'(1) &= -1 < 0 && \Rightarrow && x^* = 1 \text{ is a stable fixed point.} \\ f'(2) &= 2 > 0 && \Rightarrow && x^* = 2 \text{ is an unstable fixed point.} \end{aligned}$$

The graph of $\dot{x}$ versus $x$ confirms these results.